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Some issues about anisotropic elastic–plastic models at finite strain

F. Sidoroff [a,*], A. Dogui [b]

[a] *Laboratoire de Tribologie et Dynamique des Systèmes - CNRS UMR 5513 - Ecole Centrale de Lyon - BP 163 - F 69130 Ecully Cedex, France*

[b] *Laboratoire de Mécanique des Solides, Ecole Nationale d'Ingénieurs de Monastir, Route de kairouan, 5019 Monastir, Tunisie, France*

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Abstract

The description of anisotropic elastic–plastic behaviour at large strain is difficult. In the rigid plastic case, anisotropic plasticity is now reasonably well understood and is based on formulating anisotropic constitutive equations in a moving frame which, correctly chosen, will ensure objectivity and which can be postulated: (i) a priori to follow in some sense the material rotations (kinematical rotating frame); (ii) a posteriori by an appropriate constitutive evolution equation (plastic spin equation). With respect to the classical continuum models this is in fact an alternative for the formulation of frame indifferent constitutive equations. This rotating frame formulation of continuum mechanics is first formally presented and applied to anisotropic elasticity to show that in this case the proper rotating frame must be used to get a true elasticity. The remaining part of the paper is then devoted to the elastic–plastic case starting from a direct extension of the formalism with derivation of the elastic constitutive equation and plastic flow rule which is best written in the so-called isoclinic configuration. Finally the kinematic plastic rotating frames will be introduced and exemplified as the appropriate choice for a correct description of an anisotropic elastic–plastic behaviour.

Keywords: Large deformations; Elasto-plasticity; Anisotropic plasticity; Large strain

1. Introduction

Anisotropic plasticity at finite strain is a difficult and controversial topic, both at the conceptual and technical level. One reason for this probably is the fact that according to the usual concepts of continuum mechanics plasticity and anisotropy are incompatible: the most simple plastic model (rigid plasticity without hardening von Mises criterion) relates the deviator of the Kirchhoff stress tensor $\boldsymbol{\tau}$ to the strain rate tensor D

$$\boldsymbol{\tau} = -p\mathbf{1} - \frac{2Y_0}{3}\frac{D}{\overline{d}}, \qquad D = \frac{3\lambda}{2}\frac{\boldsymbol{\tau}^D}{\overline{\tau}} \tag{1}$$

* Corresponding author.
E-mail address: Francois.Sidoroff@ec-lyon.fr (F. Sidoroff).

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where $\overline{\tau}$ and $\overline{d}$ respectively are the von Mises equivalent stress and strain rate, λ is the plastic multiplicator and Y_0 denotes the tensile yield stress ($\overline{\tau} = Y_0$).

This in fact defines a highly non-linear viscous fluid. Indeed plasticity refers to a fluid-like behaviour. But according to Noll's definition (Truesdell and Noll, 1965), a fluid is isotropic. A plastic material therefore should be isotropic. Anisotropic plasticity nevertheless does exist but its construction implies revisiting the very definition of a simple material and its constitutive relation.

The rotating frame concept, initially introduced for the description of anisotropic rigid plasticity, provides such an approach. The purpose of the present work is to discuss this formulation and discuss its application to elastic–plastic anisotropic behaviour. Attention will be restricted to initial anisotropy. These ideas may also be useful for the description of induced or evolutive anisotropy but this is another – still more difficult – problem. For the sake of simplicity we shall also limit our presentation to prefect plasticity without hardening but the extension to classical hardening models (isotropic or kinematic) is straightforward.

2. Rotating frame formulation

2.1. Rigid plasticity

The constitutive equation for a standard rigid-plastic material is

$$D = \lambda \frac{\partial f}{\partial \boldsymbol{\tau}}, \quad f(\boldsymbol{\tau}) = 0 \tag{2}$$

with an isotropic yield function f. This formulation however does not work in the anisotropic case because objectivity is not satisfied with a non-isotropic yield function. The standard procedure consisting to switch to a Lagrangian formulation relating the time derivative of the Green–Lagrange strain to the second Piola–Kirchhoff stress does not work either due to an unphysical coupling with the initial configuration (Dogui and Sidoroff, 1986).

It is now generally accepted that the appropriate formulation is to extend Eq. (2) in

$$\overline{D} = \lambda \frac{\partial f}{\partial \overline{\boldsymbol{\tau}}}, \quad f(\overline{\boldsymbol{\tau}}) = 0, \quad D = Q\overline{D}Q^{\mathrm{T}}, \quad \boldsymbol{\tau} = Q\overline{\boldsymbol{\tau}}Q^{\mathrm{T}} \tag{3}$$

where $\overline{D}$ and $\overline{\boldsymbol{\tau}}$ are the rotated strain rate and stress tensor in a configuration $\overline{C}(t)$ (the so-called rotating frame) obtained from the current configuration $C(t)$ by the rotation $Q^{\mathrm{T}}(t)$ (Fig. 1) (Dogui and Sidoroff, 1984, 1987; Dafalias, 1983; Haupt and Tsakmakis, 1986; Paulun and Pecherski, 1985). Obviously this will result in an objective constitutive equation if $\overline{C}(t)$ is invariant in a change of frame

$$F \to F' = qF \Rightarrow Q \to Q' = qQ \tag{4}$$

where $q(t)$ is an arbitrary rotation history. Indeed in this case $\overline{D}$ and $\overline{\boldsymbol{\tau}}$ will be invariant in Eq. (4) therefore ensuring objectivity of the rotated constitutive equation (3).

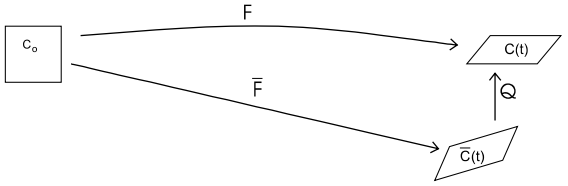

Fig. 1. Rotating frame.

It is easily shown that this condition will be true if and only if

$$\dot{Q} = WQ - Q\overline{W}, \quad W = (\dot{F}F^{-1})^{\mathrm{A}} \tag{5}$$

where W is the spin tensor, while $\overline{W}$ is invariant in Eq. (4) $\overline{W} \to \overline{W}' = \overline{W}$.

For example, the Zaremba–Jaumann corotational frame is defined by $\overline{W} = 0$

$$\dot{Q} = WQ, \quad \overline{W} = 0 \tag{6}$$

while Naghdi's proper rotation frame is defined by the rotation tensor obtained from the polar decomposition of F

$$Q = R, \quad F = RU = VR \tag{7}$$

Another approach consists in postulating a special constitutive equation for $\overline{W}$

$$\dot{Q} = WQ - Q\overline{W}(\overline{D}) \tag{8}$$

resulting in the so-called plastic spin equation where $\overline{W}(\overline{D})$ usually is postulated from representation theorems (Dafalias, 1984, 1985). We are however here primarily interested in the kinematically defined rotating frame.

2.2. Rotating frame formulation for simple materials

In the isotropic case the general constitutive equation for a simple material (Truesdell and Noll, 1965) can be written as

$$\boldsymbol{\tau} = \mathbf{T}\{C_t^t, B(t)\} \tag{9}$$

where $\mathbf{T}$ is an isotropic functional depending on the present left Cauchy–Green tensor and on the past history of the right relative Cauchy–Green tensor. Again this formulation can be extended in an anisotropic objective constitutive equation through the use of a rotated configuration $\overline{C}$

$$\overline{\boldsymbol{\tau}}(t) = \mathbf{T}\left\{\overline{C}_t^t, \overline{B}(t)\right\} \tag{10}$$

$$\dot{Q}(t) = WQ - Q\mathbf{W}\left\{\overline{C}_t^t, \overline{B}(t)\right\} \tag{11}$$

introducing together with the anisotropic response functional $\mathbf{T}$ a rheological spin function $\mathbf{W}$. Of course Eq. (10) reduces to Eq. (9) whatever $\mathbf{W}$ may be in the isotropic case. In most cases this rheological spin functional will be assumed to be a linear function of the rotated strain rate tensor

$$\overline{W} = \mathbf{K}(\overline{\text{state}})\left[\overline{D}\right] \tag{12}$$

where $\mathbf{K}$ is a linear operator depending on the state variables which of course will here be defined in the rotated configuration $\overline{C}$.

Again the corotational frame is defined by $\mathbf{K} = 0$, while Naghdi's proper rotation frame can be shown to be obtained from

$$\overline{W} = \frac{1}{V_{\mathrm{I}}V_{\mathrm{II}} - V_{\mathrm{III}}}\left\{V_{\mathrm{I}}V_{\mathrm{II}}\left(\overline{D}\,\overline{V}^{-1} - \overline{V}^{-1}\overline{D}\right) + \overline{B}\,\overline{D}\,\overline{V} - \overline{V}\,\overline{D}\,\overline{B}\right\} = \mathbf{R}\left(\overline{B}\right)\left[\overline{D}\right] \tag{13}$$

where V_{I}, V_{II} and V_{III} are the fundamental invariants of V or $\overline{V}$. These are two examples of kinematic rotating frames which are defined directly from the deformation history without referring to some special structure of the constitutive model. They can therefore be used for any kind of material and are useful when

no physical information is available on this rheological spin, which unfortunately is usually the case. Some other kinematical rotating frames may be defined (Dogui, 1988). These are our primary interest here.

It should also be noted that this is not a new idea and the objective formulation of constitutive equations in the corotational frame has been advocated quite a long time ago by Anglès d'Auriac (the so-called rheological frame) (Anglès d'Auriac, 1970). In particular an interesting model for incompressible viscoelastic fluids is defined by the hereditary integral

$$\overline{\boldsymbol{\tau}} = -p\mathbf{1} + \int_{-\infty}^{t} K(t-\tau)\overline{D}(\tau)\,\mathrm{d}\tau \tag{14}$$

2.3. Rotating frame formulation for hyperelasticity

An isotropic hyperelastic material is defined by and isotropic energy function $w(B)$ with

$$\boldsymbol{\tau} = 2B\frac{\partial w}{\partial B} \tag{15}$$

Its anisotropic rotating frame formulation (RFF) counterpart therefore will be defined by an anisotropic energy function $w(\overline{B})$. Derivation of the corresponding constitutive equation will follow from the no-dissipation condition

$$\phi = \dot{w} - \boldsymbol{\tau} : D = \dot{w} - \overline{\boldsymbol{\tau}} : \overline{D} = 0 \tag{16}$$

Expression of $\dot{w}$ requires the evaluation of $\dot{\overline{B}}$ which using Eqs. (8) and (12) is

$$\dot{\overline{B}} = \dot{Q}^{\mathrm{T}}BQ + Q^{\mathrm{T}}B\dot{Q} + Q^{\mathrm{T}}\dot{B}Q = 2\left[Q^{\mathrm{T}}B\left(WQ - Q\overline{W}\right) + Q^{\mathrm{T}}BL^{\mathrm{T}}Q\right]^{\mathrm{S}} = 2\left[\overline{B}\,\overline{D} - B\mathbf{K}\left(\overline{B}\right)[D]\right]^{\mathrm{S}}$$

where A^{S} denotes the symmetric part of any tensor A

$$\dot{w} = \frac{\partial w}{\partial \overline{B}} : \dot{\overline{B}} = 2\frac{\partial w}{\partial \overline{B}} : \left(\overline{B}\,\overline{D} - \overline{B}\mathbf{K}\left[\overline{D}\right]\right) = 2\left(\overline{B}\frac{\partial w}{\partial \overline{B}} - \mathbf{K}^{\dagger}\left[\overline{B}\frac{\partial w}{\partial \overline{B}}\right]\right) : \overline{D}$$

The constitutive equation then follows from Eq. (16) as

$$\overline{\boldsymbol{\tau}} = \overline{B}\frac{\partial w}{\partial \overline{B}} + \frac{\partial w}{\partial \overline{B}}\overline{B} - \mathbf{K}^{\dagger}\left[\overline{B}\frac{\partial w}{\partial \overline{B}} - \frac{\partial w}{\partial \overline{B}}\overline{B}\right] \tag{17}$$

which obviously reduces to Eq. (15) in the isotropic case. Since the only state variable in this case is $\overline{B}$ then the operator $\mathbf{K}$ and its adjoint $\mathbf{K}^{\dagger}$ also depends on $\overline{B}$ and so finally does $\overline{\boldsymbol{\tau}}$, so that finally RFF hyperelastic material is characterised by a constitutive equation $\overline{\boldsymbol{\tau}}(\overline{B})$.

This however does not in general correspond to an admissible elastic law, unless the differential equation (5) defines an holonomic Q. In general a closed cycle in the F space will correspond to a non-closed cycle in the Q space resulting in a non-closed stress cycle, which is not admissible for elasticity. This in particular is a well-known fact for the corotational frame.

The only holonomic rotating frame apparently is Naghdi's frame with $Q = R$. In this case $\overline{B} = C$ so that $w = w(C)$ and standard anisotropic hyperelasticity is obtained. Some efforts are however required to obtain the standard hyperelastic law from Eq. (17). This is more easily obtained from the following relation:

$$\overline{\boldsymbol{\tau}} = 2\overline{V}\frac{\partial w}{\partial \overline{B}}\overline{V} - (\mathbf{K} - \mathbf{R})^{\dagger}\left[\frac{\partial w}{\partial \overline{B}}\overline{B} - \overline{B}\frac{\partial w}{\partial \overline{B}}\right] \tag{18}$$

where the operator $\mathbf{R}$ defined in Eq. (13) is associated to Naghdi's frame. This formulation which is an alternative to Eq. (17) directly shows that if $Q = R$ then $\mathbf{K} - \mathbf{R} = 0$ so that Eq. (18) reduces to the classical hyperelastic anisotropic law

$$\overline{\tau} = R\tau R^{\mathrm{T}} = 2\overline{V}\frac{\partial w}{\partial \overline{B}}\overline{V} = 2U\frac{\partial w}{\partial C}U \tag{19}$$

3. Elasto-plasticity

3.1. Basic formulation

The fundamental elastic–plastic model is, in the isotropic case, well established. The starting points are the multiplicative decomposition $F = F^{\mathrm{e}}F^{\mathrm{p}}$ (Fig. 2) and the assumption that the elastic energy is a function of the elastic left Cauchy–Green tensor. The dissipation is then easily obtained as

$$\phi = \left(\tau - 2B^{\mathrm{e}}\frac{\partial w}{\partial B^{\mathrm{e}}}\right) : D + 2B^{\mathrm{e}}\frac{\partial w}{\partial B^{\mathrm{e}}} : R^{\mathrm{e}}D^{\mathrm{p}}R^{\mathrm{e}^{\mathrm{T}}} \tag{20}$$

which gives the elastic law

$$\tau = 2B^{\mathrm{e}}\frac{\partial w}{\partial B^{\mathrm{e}}} \tag{21}$$

and shows that τ and $R^{\mathrm{e}}D^{\mathrm{p}}R^{\mathrm{e}^{\mathrm{T}}}$ are the appropriate conjugate stress and plastic strain rate, leading to the following plastic evolution law

$$R^{\mathrm{e}}D^{\mathrm{p}}R^{\mathrm{e}^{\mathrm{T}}} = \lambda\frac{\partial f}{\partial \tau}, \qquad f(\tau) \leqslant 0 \tag{22}$$

where f is the usual yield function. This formulation however does not work for an anisotropic material, i.e. if the elastic energy $w(B^{\mathrm{e}})$ and/or the yield function $f(\tau)$ are not isotropic function. In this case the RFF can be used (Fig. 3) with an elastic energy $w(\overline{B}^{\mathrm{e}})$.

$$F = Q\overline{F}^{\mathrm{e}}F^{\mathrm{p}} = QF, \quad \overline{B}^{\mathrm{e}} = \overline{F}^{\mathrm{e}}\overline{F}^{\mathrm{e}^{\mathrm{T}}} = Q^{\mathrm{T}}B^{\mathrm{e}}Q = \overline{F}C^{\mathrm{p}^{-1}}\overline{F}^{\mathrm{T}} \tag{23}$$

The time derivative of B^{e} is evaluated as

$$\dot{\overline{B}}^{\mathrm{e}} = \left(\dot{\overline{F}}\overline{F}^{-1}\right)B^{\mathrm{e}} + B^{\mathrm{e}}\left(\overline{F}^{-1^{\mathrm{T}}}\dot{\overline{F}}^{\mathrm{T}}\right) + \overline{F}\left(C^{\mathrm{p}^{-1}}\right)^{\bullet}\overline{F}^{\mathrm{T}}$$

The time derivative of $\overline{F}$ is obtained from Eq. (8) just as in the elastic case (Section 2.2)

$$\dot{\overline{F}}\overline{F}^{-1} = \overline{D} - \overline{W}(\overline{D})$$

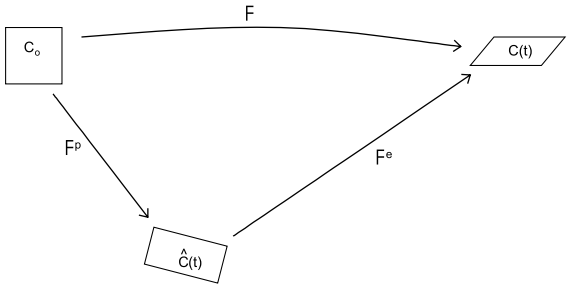

Fig. 2. Elastic–plastic deformations.

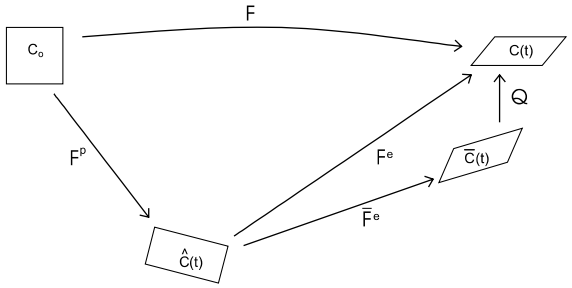

Fig. 3. Rotating frame for elastic–plastic deformations.

while the plastic term is classically expressed in terms of the plastic strain rate, finally resulting in the following expression

$$\overline{B}^{\mathrm{e}} = 2\left\{\left(\overline{D} - \overline{W}\right)B^{\mathrm{e}} + \overline{F}^{\mathrm{e}} D^{\mathrm{p}} \overline{F}^{\mathrm{e}^{\mathrm{T}}}\right\}^{\mathrm{S}}$$

This allows the evaluation of $\dot{w}$ and of the dissipation as

$$\left\{\overline{\boldsymbol{\tau}} - 2\left(\frac{\partial w}{\partial \overline{B}^{\mathrm{e}}}\overline{B}^{\mathrm{e}} - \mathbf{K}^{\dagger}\left[\frac{\partial w}{\partial \overline{B}^{\mathrm{e}}}\overline{B}^{\mathrm{e}}\right]\right)\right\} : \overline{D} + 2\overline{F}^{\mathrm{e}^{\mathrm{T}}}\frac{\partial w}{\partial \overline{B}^{\mathrm{e}}}\overline{F}^{\mathrm{e}} : D^{\mathrm{p}}$$

from which follows the elastic law

$$\overline{\boldsymbol{\tau}} = \frac{\partial w}{\partial \overline{B}^{\mathrm{e}}}\overline{B}^{\mathrm{e}} + \overline{B}^{\mathrm{e}}\frac{\partial w}{\partial \overline{B}^{\mathrm{e}}} - \mathbf{K}^{\dagger}\left[\frac{\partial w}{\partial \overline{B}^{\mathrm{e}}}\overline{B}^{\mathrm{e}} - \overline{B}^{\mathrm{e}}\frac{\partial w}{\partial \overline{B}^{\mathrm{e}}}\right] \tag{24}$$

and the tensor $\boldsymbol{\tau}^{\mathrm{p}}$

$$\phi = \boldsymbol{\tau}^{\mathrm{p}} : D^{\mathrm{p}}, \quad \boldsymbol{\tau}^{\mathrm{p}} = 2\overline{F}^{\mathrm{e}^{\mathrm{T}}}\frac{\partial w}{\partial \overline{B}^{\mathrm{e}}}\overline{F}^{\mathrm{e}} \tag{25}$$

which is the thermodynamic force associated to D^{p} and from which must therefore be expressed the plastic yield condition. These relations of course reduce to Eq. (21) with $\boldsymbol{\tau} = \boldsymbol{\tau}^{\mathrm{p}}$ in the elastic isotropic case.

3.2. The isoclinic configuration

The formulation presented above is essentially based on the tensor $\overline{B}^{\mathrm{e}}$ associated to the rotated elastic deformation

$$\overline{F}^{\mathrm{e}} = Q^{\mathrm{T}} F^{\mathrm{e}} = \overline{V}^{\mathrm{e}}\overline{R}^{\mathrm{e}} \tag{26}$$

On the other hand, the plastic evolution equation will provide the plastic rate of deformation, therefore leaving indeterminate the orientation of the intermediate configuration $\hat{C}$ or equivalently the elastic rotation $\overline{R}^{\mathrm{e}}$. The simplest way to override this indeterminacy is to use as intermediate configuration the configuration $\overline{C}^{\mathrm{e}}$ obtained from $\overline{C}$ by unloading without rotation. Eq. (25) is unchanged while Eq. (26) becomes

$$\phi = \overline{\boldsymbol{\tau}}^{\mathrm{p}}\overline{D}^{\mathrm{p}}, \quad \overline{\boldsymbol{\tau}}^{\mathrm{p}} = \overline{R}^{\mathrm{e}}\boldsymbol{\tau}^{\mathrm{p}}\overline{R}^{\mathrm{e}^{\mathrm{T}}} = 2\overline{V}^{\mathrm{e}}\frac{\partial w}{\partial \overline{B}^{\mathrm{e}}}\overline{V}^{\mathrm{e}} \tag{27}$$

$$\overline{D}^{\mathrm{p}} = \left(\dot{\overline{F}}^{\mathrm{p}}\overline{F}^{\mathrm{p}^{-1}}\right)^{\mathrm{S}} = \overline{R}^{\mathrm{e}} D^{\mathrm{p}} \overline{R}^{\mathrm{e}^{\mathrm{T}}} \tag{28}$$

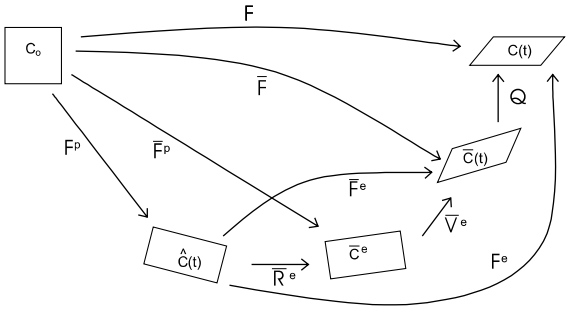

Fig. 4. The isoclinic configuration.

The plastic evolution equation will then be simply obtained by the usual normality rule from the yield function $f(\overline{\tau}^{p})$

$$\overline{D}^{p} = \lambda \frac{\partial f}{\partial \overline{\tau}^{p}} \tag{29}$$

Eqs. (28) and (30) are the constitutive equations of the RFF of elasto-plasticity which corresponds to an objective constitutive model which coincides with the isotropic model (21) when the two functions w and f are isotropic but remains valid in the anisotropic case. Such a model is defined by these two functions but also by the choice of the rotating frame.

It should be noted that this approach naturally introduces the special stress free configuration $\overline{C}^{e}$, which in analogy with Mandel's work (Mandel, 1971, 1981) will be called the isoclinic configuration (Fig. 4) and which corresponds to the following decomposition of F

$$F = Q\overline{V}^{e}\overline{F}^{p}$$

From a practical point of view, this model introduces as internal variables the rotation Q and the plastic deformation tensor $\overline{F}^{p}$. For a prescribed evolution of the total strain gradient F the corresponding evolution of the rotation Q directly follows from Eq. (11) and from the choice of the rotating frame while the evolution of $\overline{F}^{p}$ is defined by Eq. (29) up to an arbitrary rotation, which in turn will be determined from the symmetry of $\overline{V}^{e}$. This procedure has in fact to be implemented in order to obtain the incremental constitutive equations to be used in numerical simulations.

4. Kinematic plastic rotating frame

4.1. Definition

The model discussed above provides a general framework for the description of anisotropic elasto-plasticity in large transformations. In the investigated standard formulation it essentially relies on

- an elastic energy function w;
- a plastic yield function with its associated flow rule f;
- a rotating frame evolution which in the case investigated here is assumed a priori from the kinematics.

This however is not sufficient to ensure a reasonable model. Indeed in order to achieve a reasonable description of finite elasto-plasticity, the two following conditions must be fullfilled.

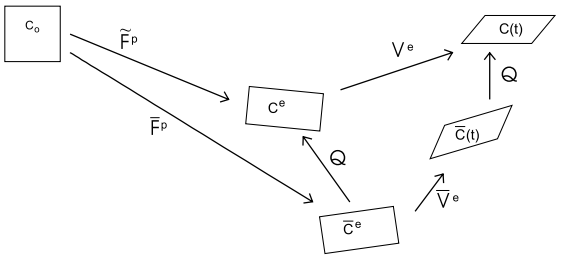

Fig. 5. Kinematic plastic rotating frame.

1. In the rigid-plastic case ($V^e \sim 1$) it must reduce to the well-known rigid-plastic model as discussed in Section 2.1.
2. It must remain hyperelastic during an elastic transformation ($f < 0$, $D^p = 0$).

These two conditions in fact express what should be expected for an elastic–plastic model: an unchanging elastic behaviour with respect to changing natural reference state (Bertram, 1998).

The basic idea to achieve this will be to use a kinematic rotating frame defined from the plastic deformation gradient $\widetilde{F}^p$ associated to the stress free configuration C^e obtained from the current configuration C by pure elastic strain without rotation (Fig. 5)

$$F = V^e \widetilde{F}^p$$

The kinematic plastic rotating frame will be obtained by substitution of $\widetilde{F}^p$ for F in the definition of the corresponding kinematic rotating frame (12).

It should be noted that with this definition the rotating frame rotation Q can at the same time (Fig. 5) be considered as

- the usual elastic rotation tensor resulting from the polar decomposition

$$\widetilde{F}^e = V^e Q = Q\overline{V}^e$$

  As discussed in Section 2.3 this is in fact the required condition to ensure a true hyperelastic behaviour.
- the standard rotating frame rotation of Section 3.1 in the rigid plastic limit ($V^e = 1, C = C^e$) in which case Fig. 5 reduces to Fig. 1.

4.2. Examples

As discussed in Section 3.2, a kinematic rotating frame is defined by Eq. (12) with a linear operator **K** depending on the $\overline{B}$ or $\overline{V}$. Accordingly the corresponding kinematic plastic-rotating frame will be defined by

$$\dot{Q} = \widetilde{W}^p Q - Q\mathbf{K}(\overline{V}^p)\left[\overline{D}^p\right], \quad \widetilde{W}^p = \left(\dot{\widetilde{F}}^p \widetilde{F}^{p^{-1}}\right)^S \tag{30}$$

In particular the corotational plastic rotating frame will be defined as

$$\dot{Q} = \widetilde{W}^p Q$$

and it is associated to the plastic Jaumann derivative defined as

$$X^{p^J} = \dot{X} - \widetilde{W}^p X + X\widetilde{W}^p$$

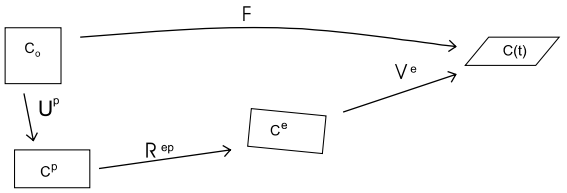

Fig. 6. Kinematic plastic proper rotation frame.

Probably more interesting is the plastic proper rotating frame defined as (Fig. 6)

$$Q = R^{ep}, \quad F = V^{e} R^{ep} U^{p}$$

with the following evolution equation

$$\dot{R}^{ep} R^{ep^{T}} = \widetilde{W}^{p} - \mathbf{R}(\widetilde{V}^{p})\left[\widetilde{D}^{p}\right]$$

where the **R** operator has been defined in Eq. (13). This probably is the most natural intrinsic decomposition of F into elastic and plastic strains.

5. Conclusion

Elastic–plastic transformations in the general anisotropic case give rise to difficult kinematical problems which are not yet entirely solved. Some of them have been discussed here and a complete consistent framework has been proposed, but it leads to a complex formulation which is still open to discussion. An effort is probably required in the next few years towards clarification and synthesis.

Fortunately, these issues – however interesting from a fundamental point of view – are of little practical importance. They disappear, in particular for small elastic strain or for elastic isotropy, and therefore for most applications.

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